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Trapped image guides are analyzed using a new method. The results agree much better with experimental data than those previously derived from a simple effective dielectric constant approach.

### Introduction

Recently, the trapped image guide was proposed as a transmission line for millimeter-wave integrated circuits.<sup>1</sup> As shown in Fig. 1(a), the structure consists of a dielectric rod placed in a metal trough so that the radiation loss is much less at a horizontal bend than in the case of a conventional image guide. The analysis presented in Reference [1] is not very accurate at lower frequencies because the fields in Regions 5 and 6 are not taken into consideration.

This paper presents a new method which predicts the characteristics of the trapped image guide much more accurately. Comparison of the numerical results with experimental data confirms the usefulness of this new method. In this paper, we discuss only the  $E^y$ -type mode.

### Analysis Procedure

The strategy is to first find an equivalent structure Fig. 1(c) of the original one in Fig. 1(a) and then apply the transverse resonance condition in the vertical direction at  $y = 0$  in Fig. 1(c) after the effect of free space  $y > 0$  is taken into account. To this end, we first apply the effective dielectric constant (EDC) method to Regions 1, 2 and 3 in Fig. 1(a). In this process, we solve an eigenvalue problem for a hypothetical structure in Fig. 1(d) in which an infinitely long vertical slab of width  $2a$  is sandwiched between two infinitely long vertical metal walls by way of air regions of width  $c$ . The problem is a two dimensional one and an exact formulation is readily derived by matching  $E_y$  and  $H_z$  at  $x = \pm a$ . Once the solution to the eigenvalue equation,  $k_x$ , is obtained, the effective dielectric constant  $\epsilon_{eI}$  of the area consisting of Regions 1, 2 and 3 is given by

$$\epsilon_{eI} = \epsilon_r - \left( \frac{k_x}{k_o} \right)^2 \quad (1)$$

Notice that the EDC method is applied to the sideward direction first in contrast to the vertical direction commonly encountered in usual EDC methods.<sup>1</sup> The reason for this becomes obvious in the next step in which the structure in Fig. 1(a) is modelled by the one in Fig. 1(b). This is a channel guide filled with a dielectric material  $\epsilon_{eI}$  up to height  $b$ .

We can now temporarily neglect the fields in the regions corresponding to Regions 5 and 6 in Fig. 1(a), and model the structure in Fig. 1(b) by that one in Fig. 1(e). Note that this structure can be solved exactly. We obtain yet another structure in Fig. 1(f). The hypothetical dielectric constant  $\epsilon_{eII}$  filling the trough up to the height  $h$  in Fig. 1(f) is chosen in such a way that the propagation constants

in the  $z$  direction in Fig. 1(e) and Fig. 1(f) are identical.

We will use the effective dielectric constant  $\epsilon_{eII}$  obtained in the above process in the final structure Fig. 1(c). Kaneki introduced an analytical method for such structures.<sup>2</sup> Use is made of the fact that the fields in the associated structure in Fig. 1(f) may be obtained from Maxwell's equations by using a scalar potential

$$\phi^e = \begin{cases} C_1 \exp(-vy) \\ C_2 \cos u(y+h) \end{cases} \cos \frac{\pi x}{2(a+c)} \quad \begin{matrix} y > 0 \\ -h < y < 0 \end{matrix} \quad (2)$$

for the dominant mode. The wavenumbers  $v$  and  $u$  and the constants  $C_1$  and  $C_2$  may be obtained from the field continuity at  $y = 0$  in Fig. 1(f). The field for  $-h < y < 0$  thus obtained is used to derive the solution to the structure for Fig. 1(c) which is considered the final solution to Fig. 1(a).

We apply the transverse resonance condition  $\vec{Z}_a + \vec{Z}_e = 0$  at  $y = 0$  in Fig. 1(c), where  $\vec{Z}_a$  is the impedance looking into air at  $y = 0$ ,  $|x| < \infty$  and  $\vec{Z}_e$  that of  $\epsilon_{eII}$  medium at  $y = 0$ ,  $|x| < a + c$ . The former,  $\vec{Z}_a$ , can be derived by computing the power  $P_a$  transmitted into air from the dielectric surface  $y = 0$  per unit length in  $z$ .

$$P_a = -\pi \omega \epsilon_o \left[ \frac{C_2 u}{\epsilon_{eII}(a+c)} \right]^2 J \sin^2(uh) \quad (3)$$

where

$$J = j \frac{(a+c)^2}{\pi} \left\{ \frac{(\pi/2)^2 + [\beta(a+c)]^2}{\sqrt{(\pi/2)^2 + p^2}} + \right. \quad (4)$$

$$\left. \frac{\pi}{2} \int_p^\infty \frac{1+e^{-2r}}{\sqrt{r^2-p^2}} \cdot \frac{[\beta(a+c)]^2 - r^2}{[(\pi/2)^2 + r^2]^2} dr \right\}$$

$$p^2 = (\beta^2 - k_o^2)(a+c)^2 \quad (5)$$

and  $\beta$  is the propagation constant in the  $z$  direction we are trying to obtain. Then  $\vec{Z}_a = P_a / |I|^2$  where  $I$  is the mode current in  $y$  due to the magnetic field at  $y = 0$ .

$\vec{Z}_e$  can be readily calculated from  $\phi^e$  for  $y < 0$  in (2), and we obtain the desired eigenvalue equation for the propagation constant  $\beta$  of Fig. 1(a).

### Numerical and Experimental Results

Some numerical results are presented in Fig. 2 along with experimental data for the  $E_{11}^y$  mode of three trapped image guides with identical dielectric rods but with different sidewall spacings. For

comparison, the characteristics of the conventional image guide ( $c = \infty$ ) with the same rod dimensions are included. It is seen that the effect of the sidewalls are more pronounced for smaller spacing  $c$  of the sidewalls. We also performed experimental measurement of the propagation constant by means of a movable probe<sup>3</sup> between 8 and 16 GHz and plotted in Fig. 2 with  $\circ$ ,  $\times$  and  $\Delta$ . It is clear that agreement between theoretical and experimental results is quite good even at lower frequencies. This was not the case in Reference 1 in which a conventional effective dielectric constant method was used after the field in Regions 5 and 6 in Fig. 1(a) were neglected.

Comparison between the present method and the previous EDC theory is given in Fig. 3 and Fig. 4. It is evident that the present method provides much more accurate results especially at lower frequencies. In fact, the previous method cannot correctly predict the dispersion characteristics at lower frequencies because the field in Regions 5 and 6 is neglected. On the other hand, such a field is incorporated in the solution process in the new method. Experimental results in Fig. 3 are obtained by the movable probe while those in Fig. 4 are reproduced from the previous work.<sup>1</sup>

Fig. 5 studies the effect of the sidewall height  $h$  on the propagation constant. Both theoretical and experimental studies indicate little difference in results due to differences in  $h$  as long as  $h$  is reasonably larger than  $b$ . In fact, it was not possible to distinguish theoretical results on the figure.

## Conclusions

The method presented here is found to provide numerical dispersion characteristics much more accurate than the one previously available for the trapped image guide structure. The method is based on the effective dielectric constant and the transverse resonance of an equivalent structure. The method is more useful in the design of the trapped image guide.

## References

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- [2] T. Kaneki, "A comparison between the channel guide and trough guide about the phase constant," *Electronics and Communications in Japan*, Vol. J59-B, pp. 211-213, March 1976.
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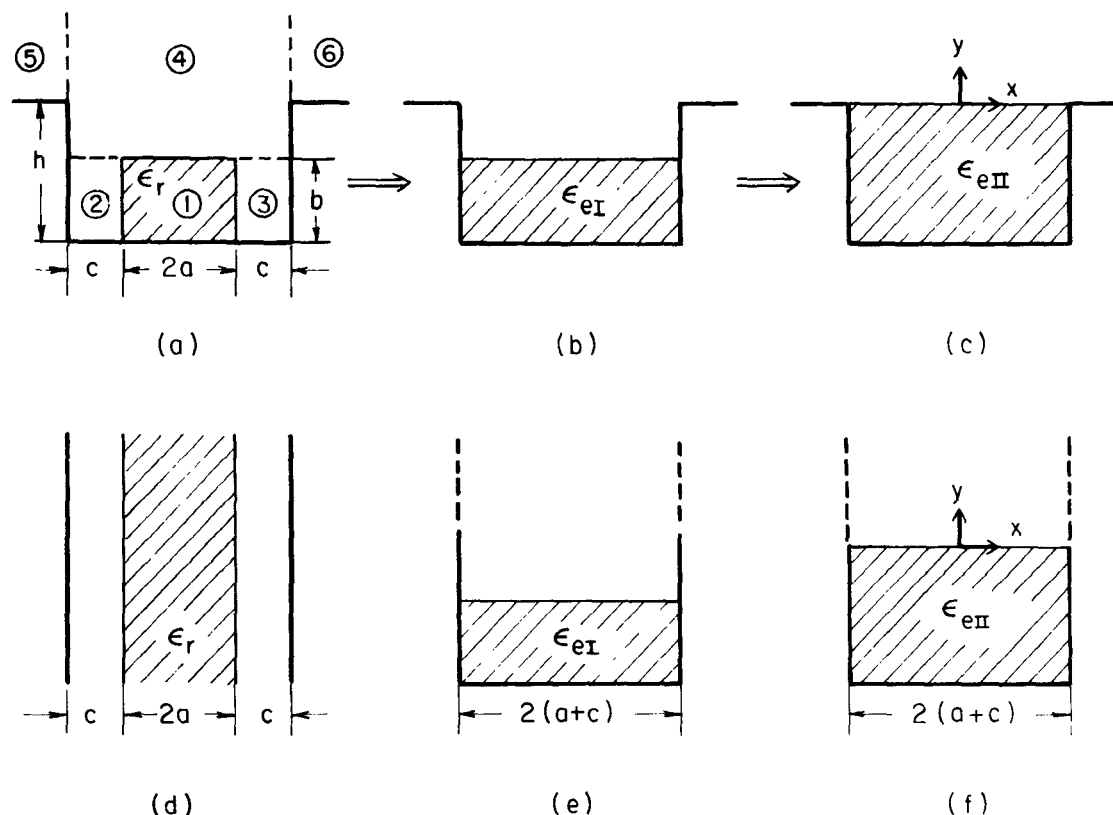


Figure 1 Cross section of the trapped image guide and its equivalent structures

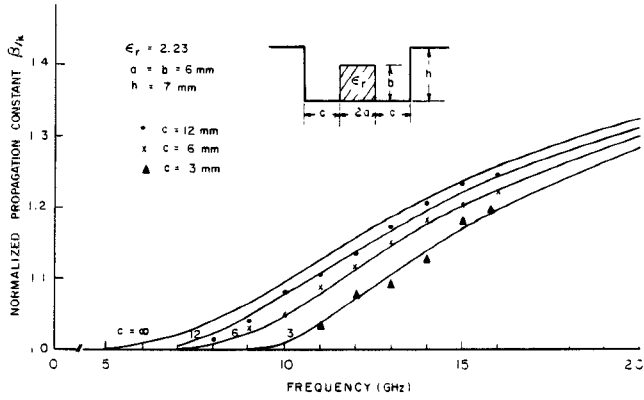


Figure 2 Theoretical and experimental dispersion characteristics.

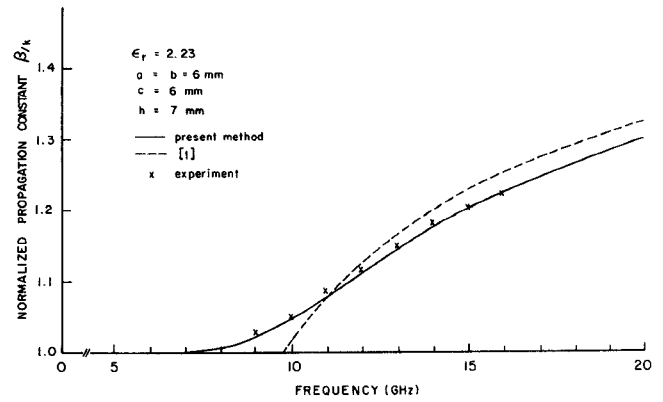


Figure 3 Comparison between the present method and reference [1].

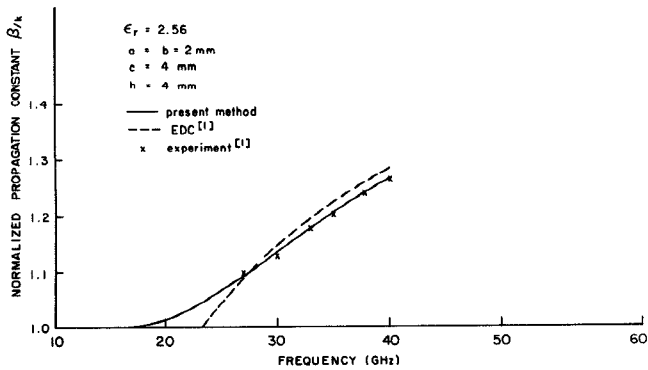


Figure 4 Comparison between the present method and reference [1].

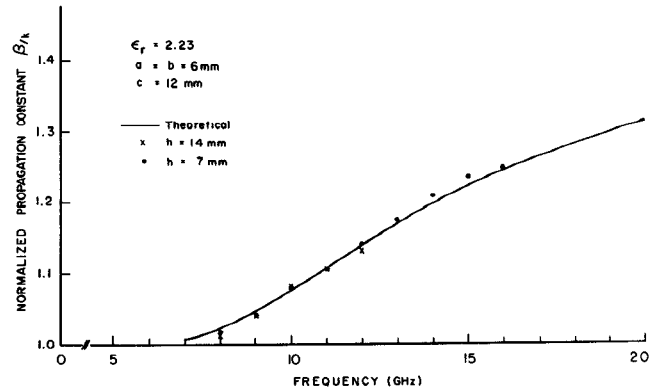


Figure 5 Effect of height of sidewalls on the dispersion characteristics.